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On the Impossibility of Obfuscating Programs

UbiCrypt Seminar & Lecture Series, 18.02.2016

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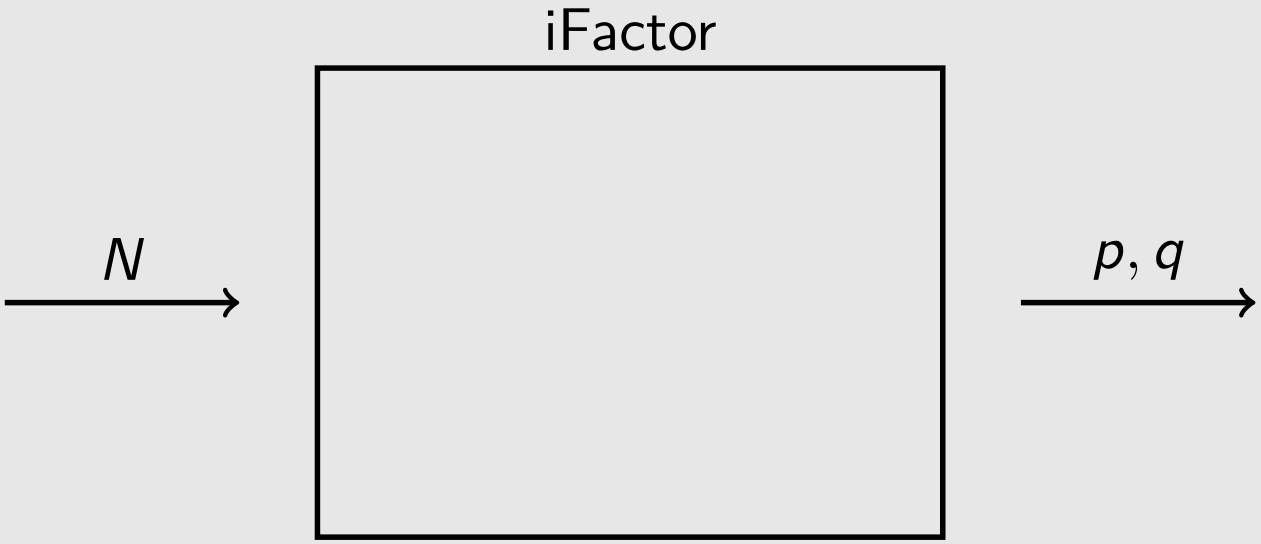
Horst Görtz Institut
für IT-Sicherheit

On the Impossibility of Obfuscating Programs

- *On the (Im)possibility of Obfuscating Programs*, CRYPTO'01
  Barak, Goldreich, Impagliazzo, Rudich, Sahai, Vadhan, Yang

Virtual Blackbox Obfuscation does not exist.

What is Virtual Blackbox Obfuscation?



Suppose, you came up with a really clever algorithm for factoring. You want to sell it without revealing your code.

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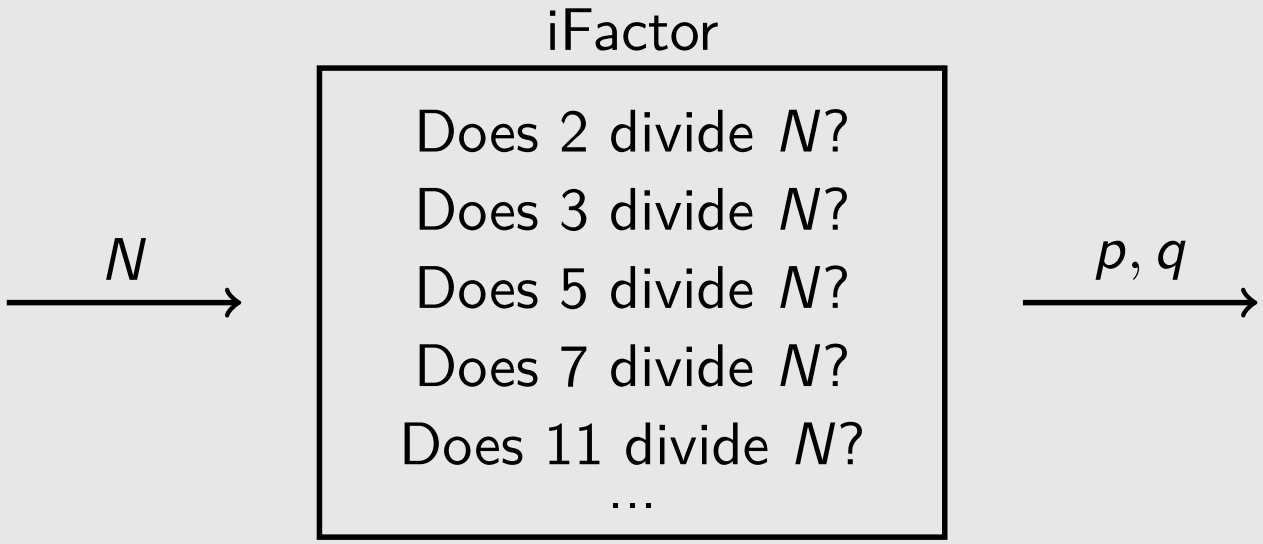

Suppose, you came up with a really clever algorithm for factoring. You want to sell it without revealing your code. (For instance, because it's actually stupid.)

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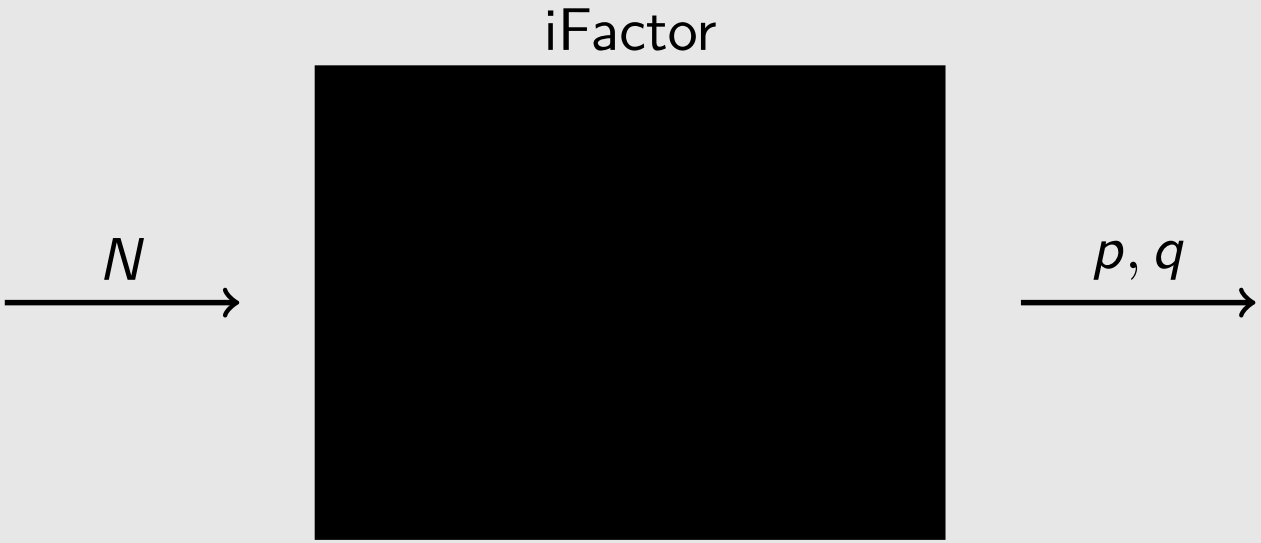

The obfuscated code should still compute the same function and should be roughly as efficient as the unobfuscated code.

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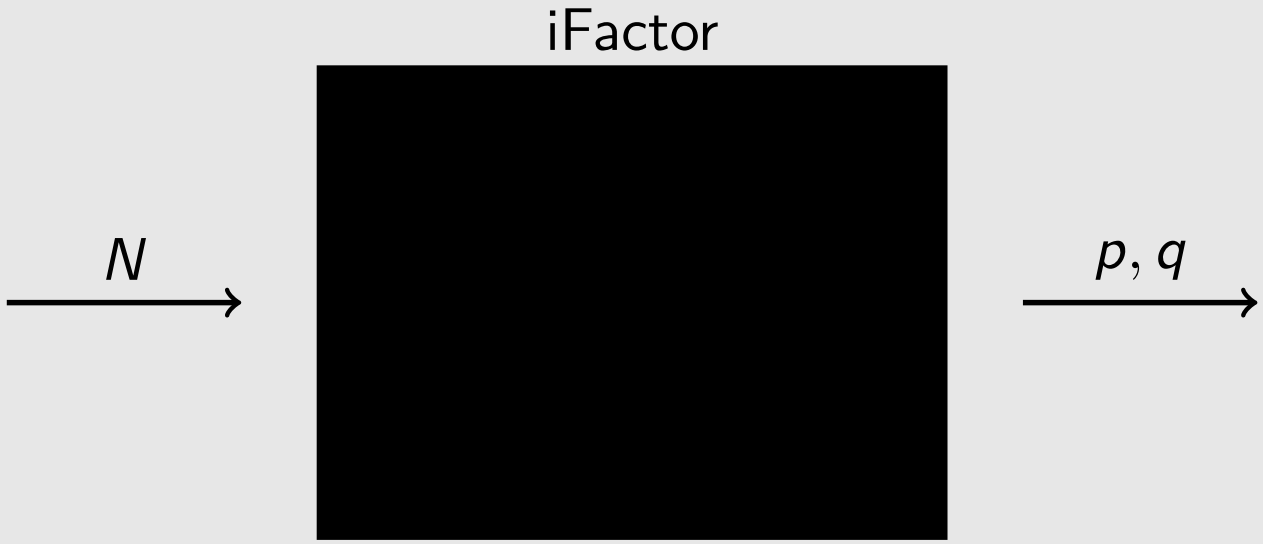

Ideally, the obfuscated code should behave like a black box, i.e. it should not leak any information beyond input/output behaviour.

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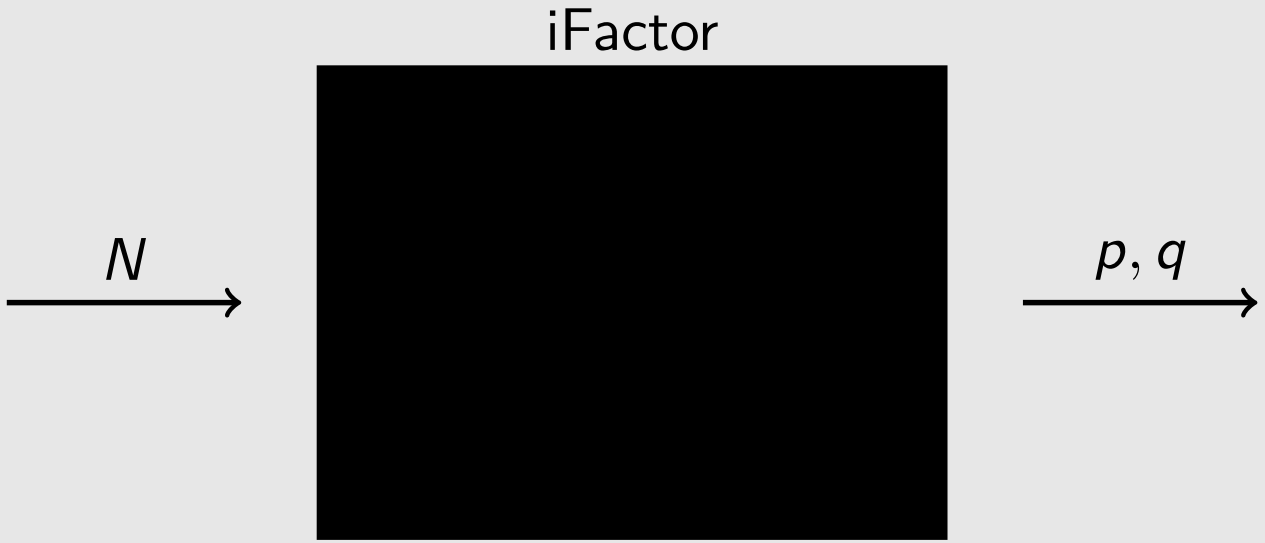

Ideally, the obfuscated code should behave like **an oracle** i.e. it should not leak any information beyond input/output behaviour.

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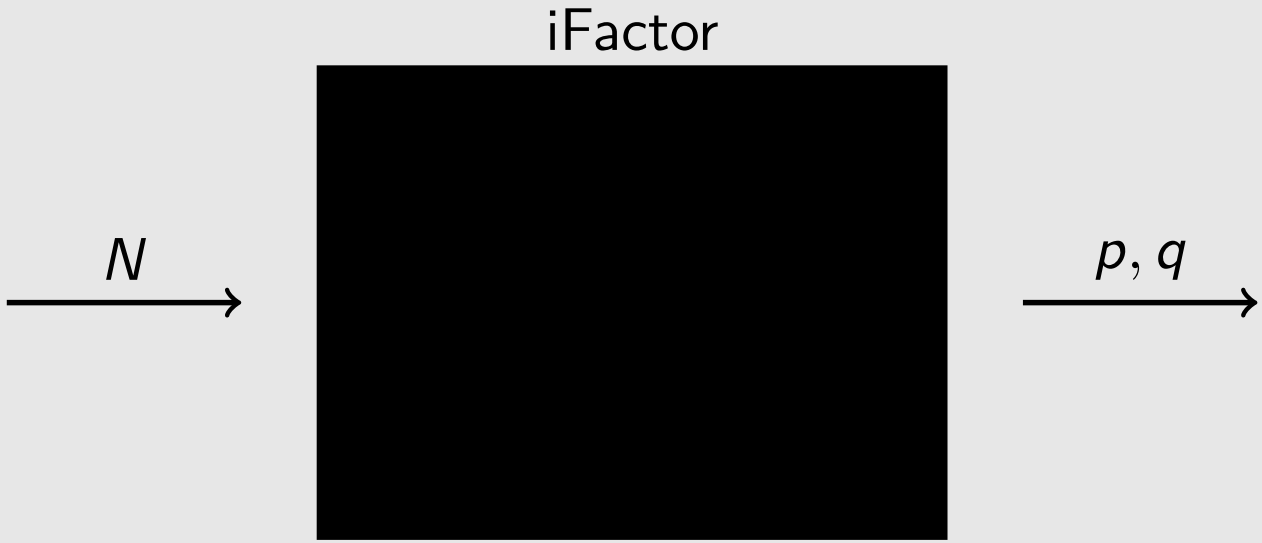

Ideally, the obfuscated code should behave like **an oracle** i.e. it should not leak any information beyond input/output behaviour.

For any efficient $\mathcal{A}$ there is an efficient $\mathcal{S}$ s.t. for all TM M:

$$\Pr[\mathcal{A}(\boxed{M}) = 1] \approx \Pr[\mathcal{S}^{M} = 1]$$

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- $\exists VBB \Rightarrow \exists OWFs \Rightarrow P \neq NP$.

Theorem: VBB O for TMs does not exist.

xkcd.com

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Proof idea:

Conceptual difference:

- Hands-on access allows one to do computations *on a function's description*, no matter how obfuscated.
- Oracle access to a function rules that out, unless the function is efficiently learnable by queries.

$$\Pr[\mathcal{A}(\boxed{M}) = 1] \approx \Pr[S^M = 1]$$

VBB "security"

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We will construct $\mathcal{A}$ and TMs, such that assuming the obfuscatability of the TMs leads to a contradiction for all $\mathcal{S}$.

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Proof idea:

$$C_{\alpha,\beta}(x) := \begin{cases} \beta & x = \alpha \\ 0^k & \textit{otherwise} \end{cases}$$
$$\alpha, \beta \leftarrow_{\$} \{0,1\}^k$$

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Actually, not computable. Suffices to let it run for a while assuming that C terminates in reasonable time.

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Consider $\mathcal{A}$ that obtains two TMs C, D encoded as (one TM called) $C\#D$ and always outputs $\mathcal{A}(C\#D) := D(C)$.

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$\mathcal{S}$ would have to query $C_{\alpha,\beta}$ (resp. Z_k) on β to tell the oracles apart.

$\Pr[\mathcal{A}(\boxed{M}) = 1] \approx \Pr[\mathcal{S}^M = 1]$ VBB "security"

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On the other hand

$$\Pr[\mathcal{A}(\boxed{Z_k\#D_{\alpha,\beta}}) = 1] \leq \text{negl}.$$

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$\mathcal{A}$ returns 0 unless $\beta = 0^k$.

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Contradiction:

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$$\overset{\text{VBB}}{\approx}$$

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