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On the Selective-Opening Security of DHIES

and other practical encryption schemes

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Selective-Opening Attacks

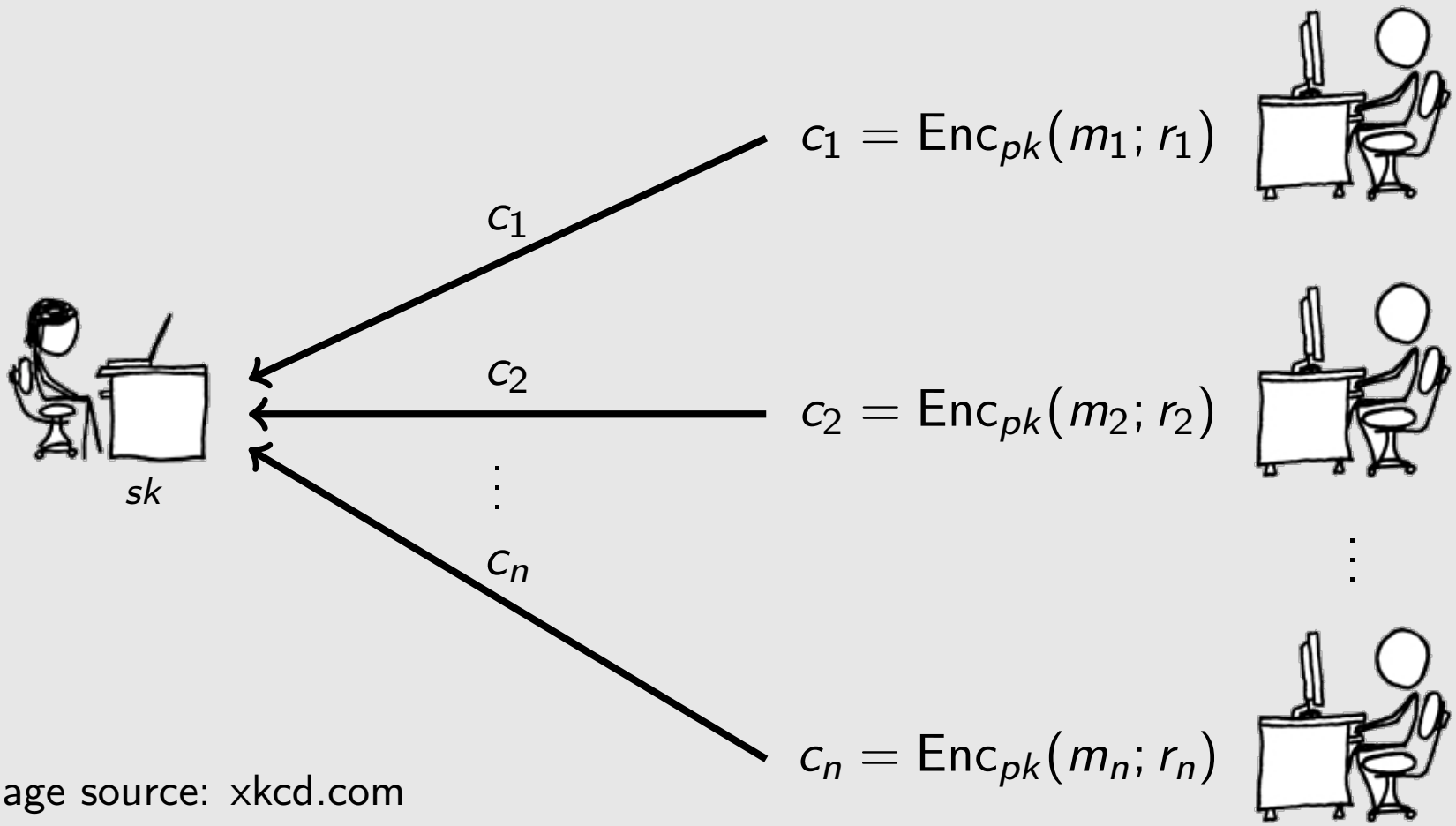

Image source: xkcd.com

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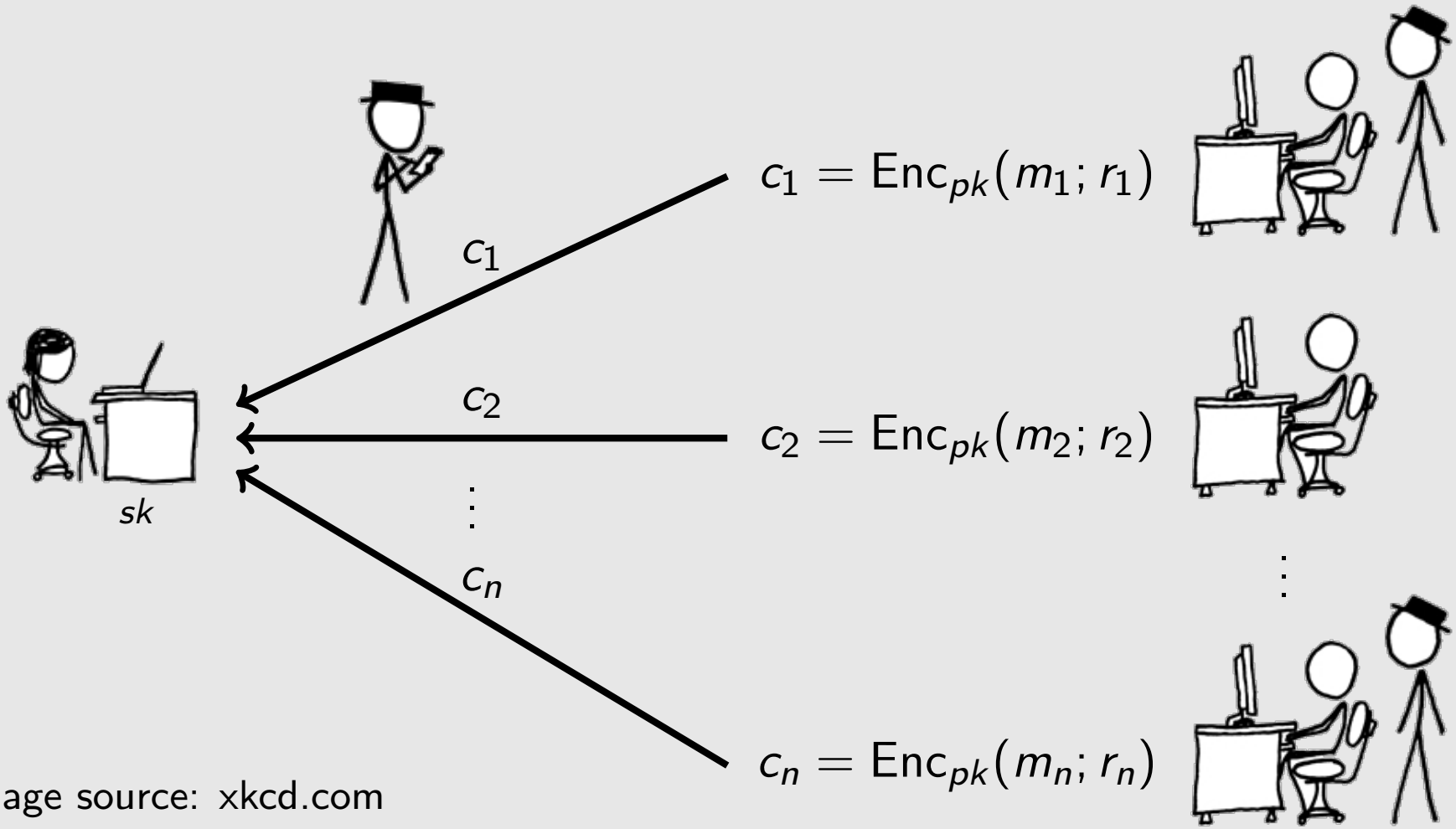

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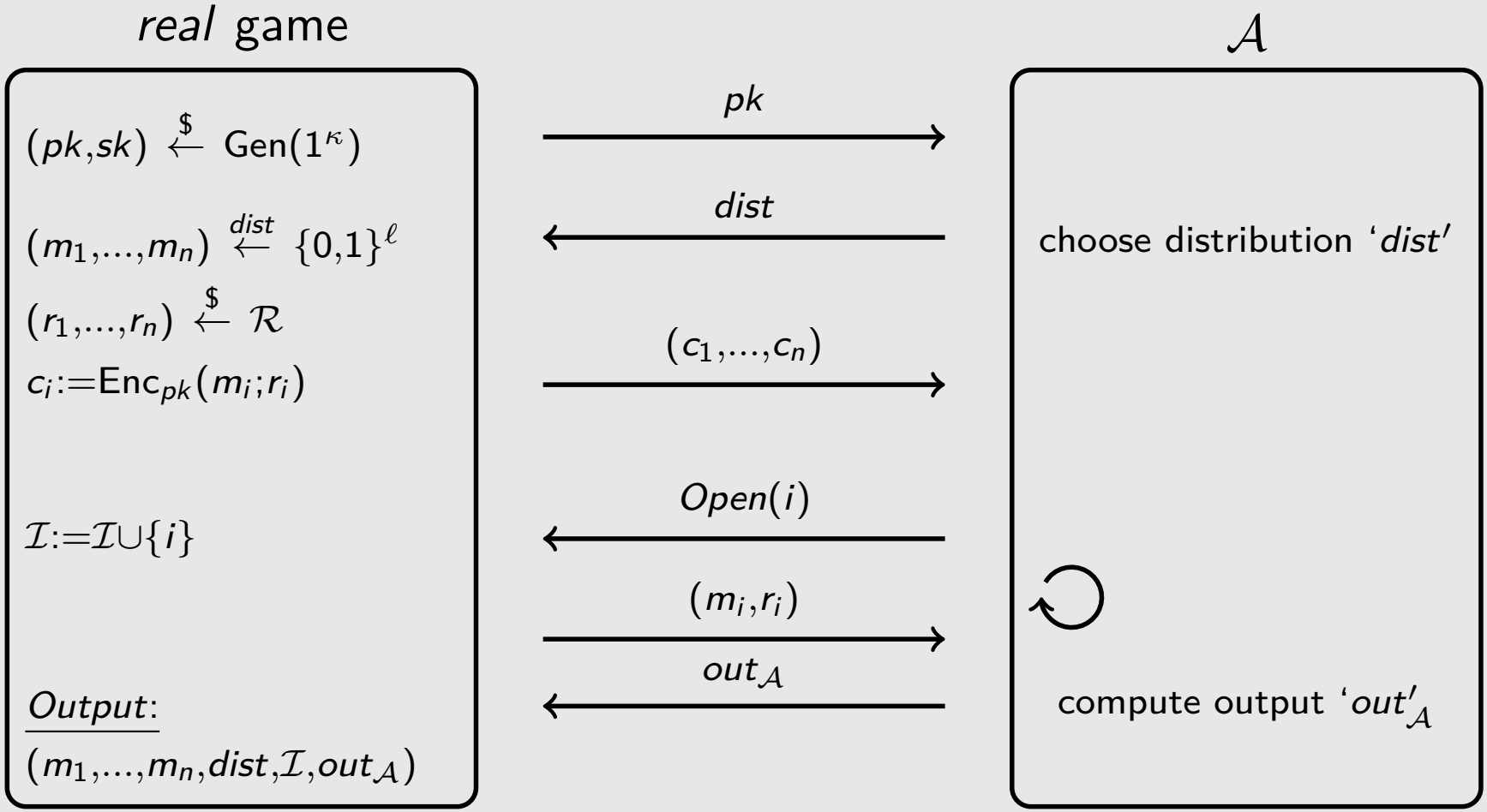
SIM-SO-CPA security definition



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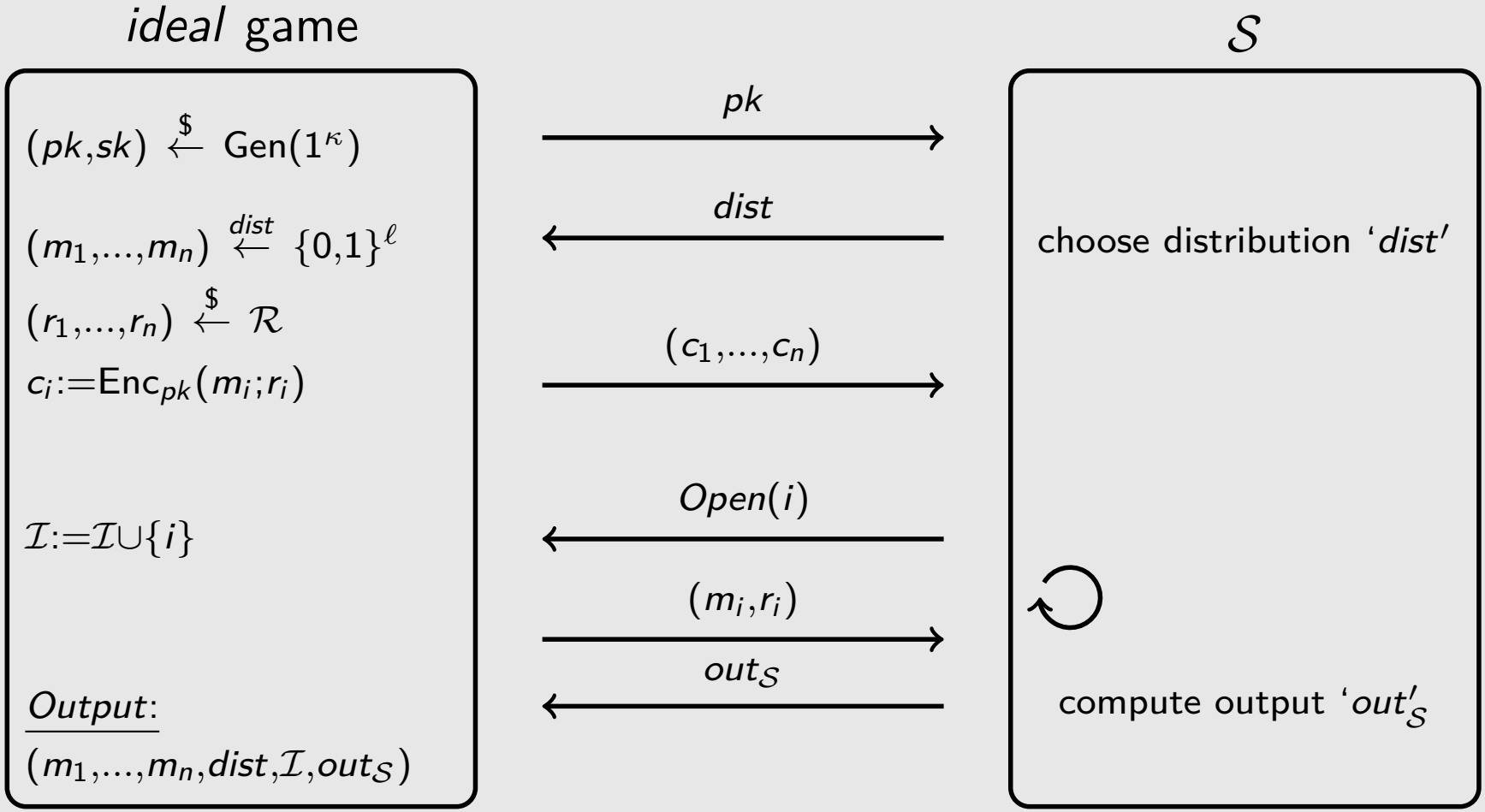

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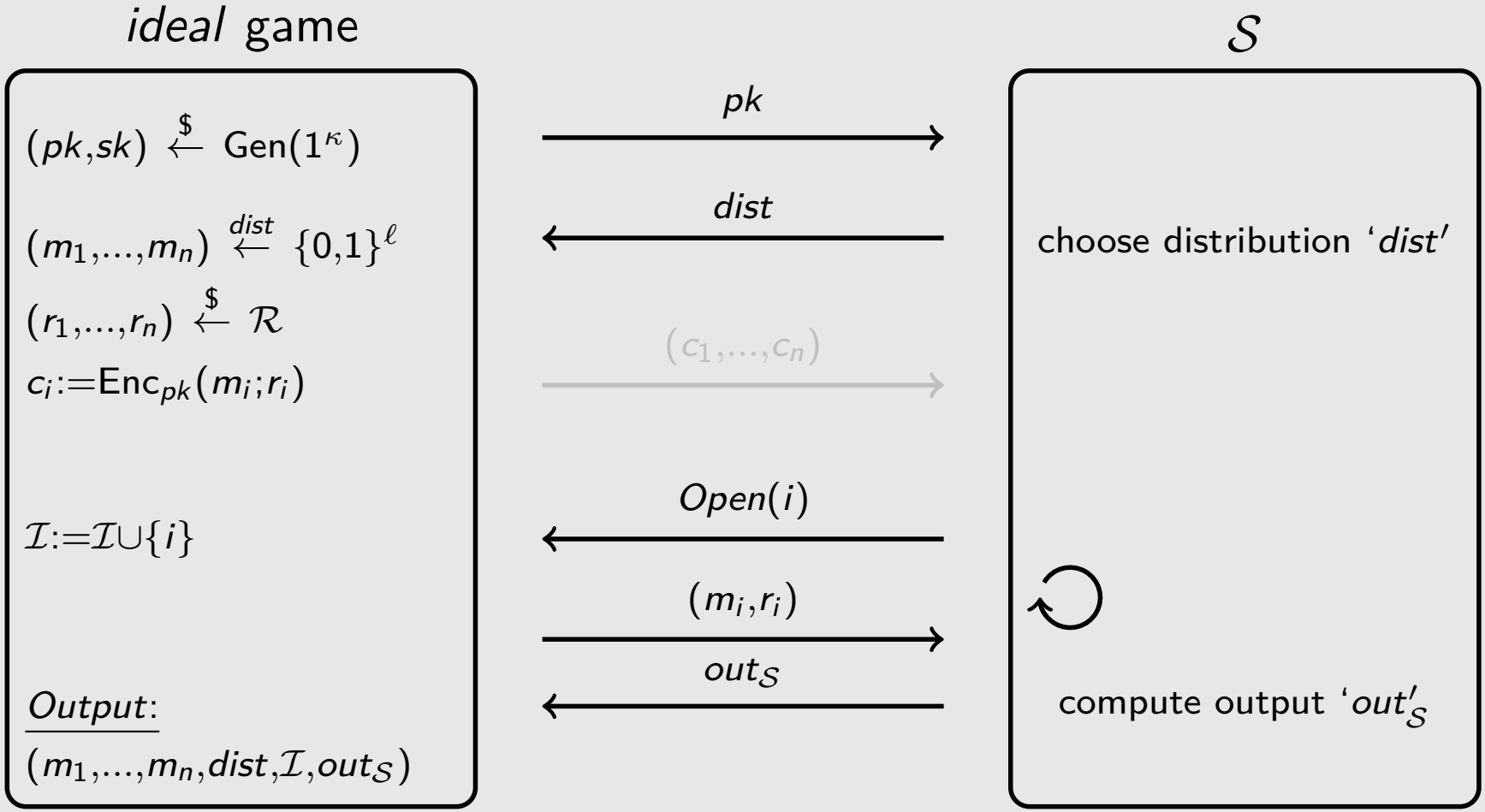

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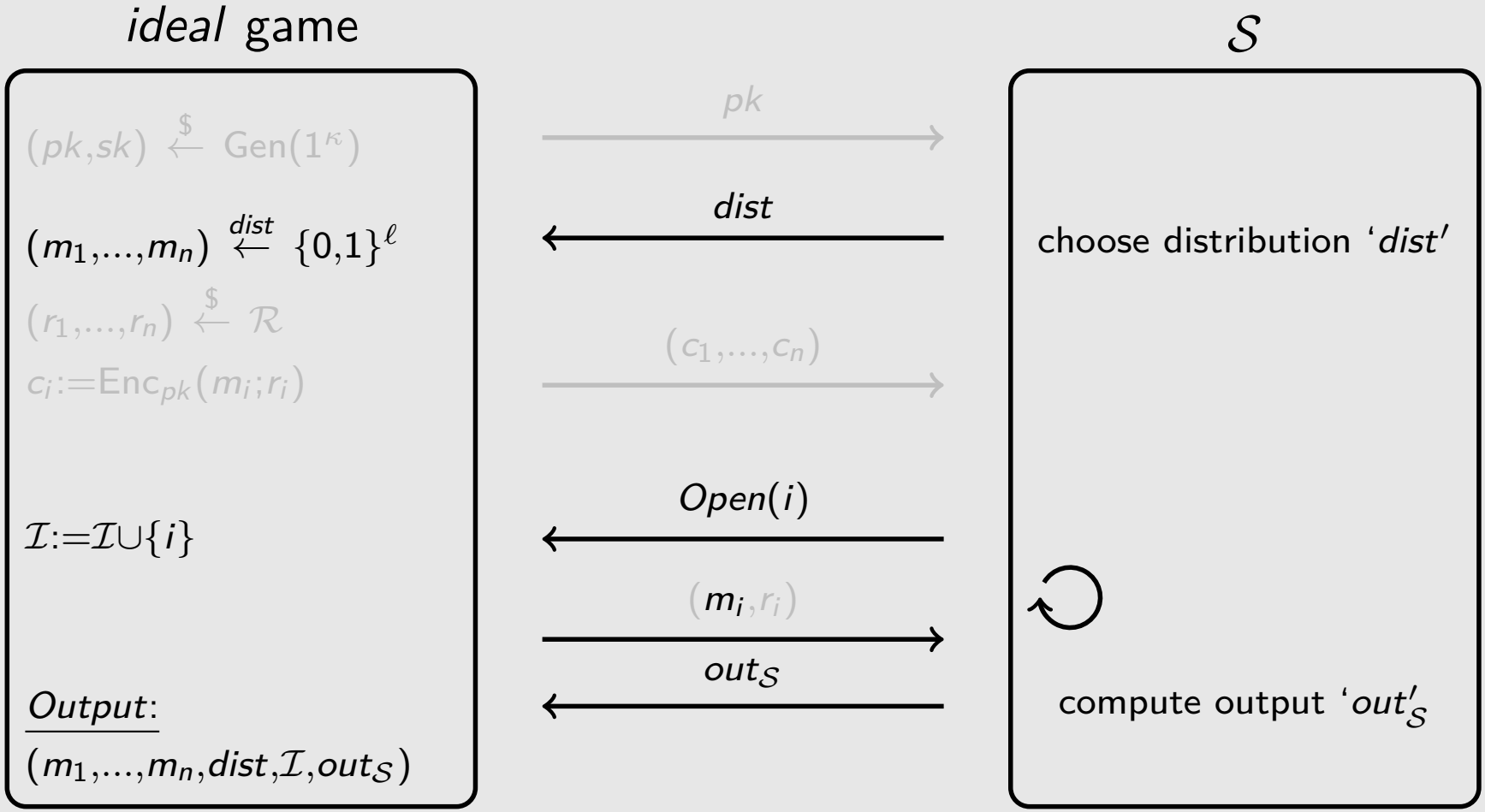

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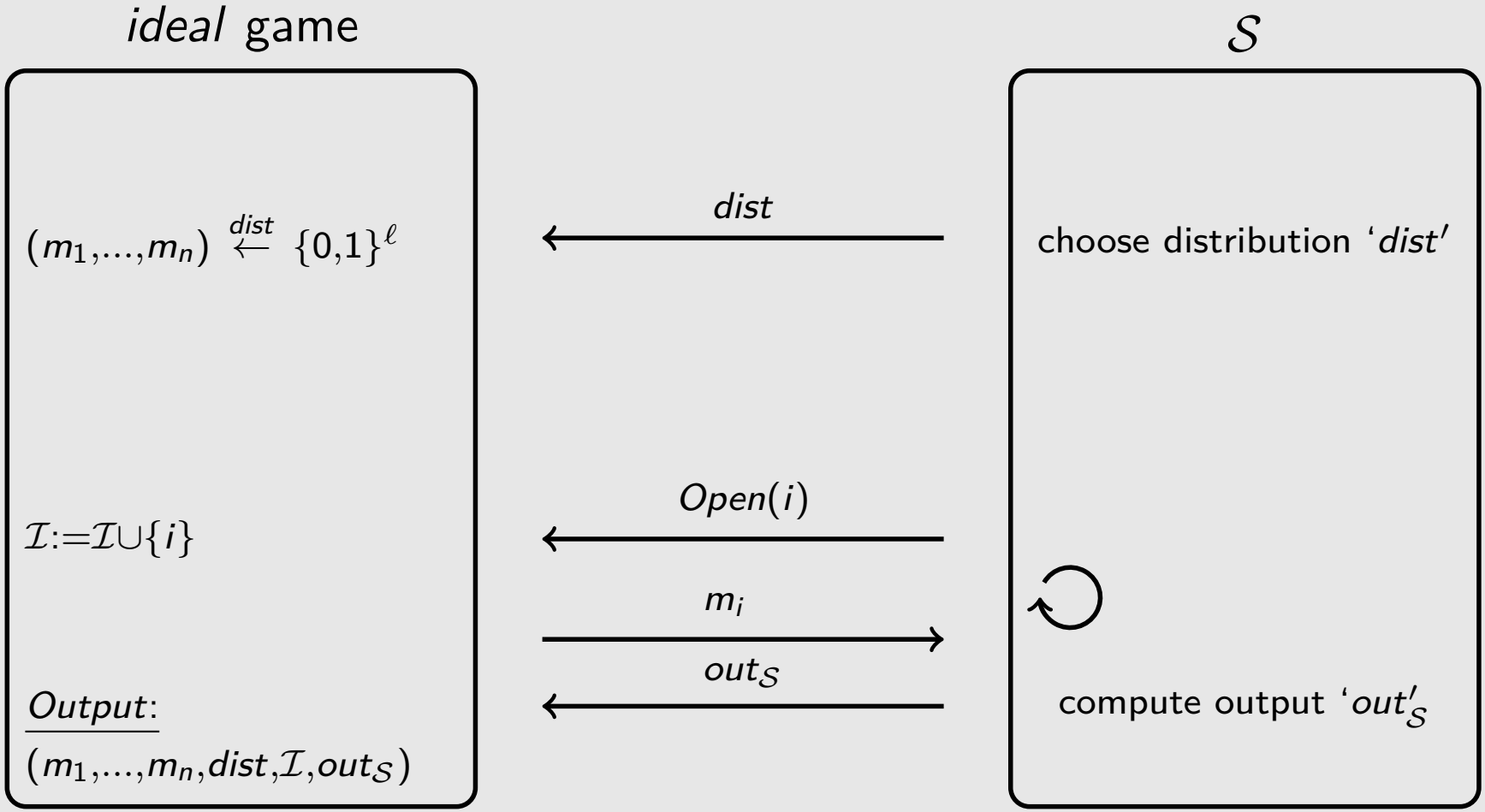

SIM-SO-CPA security definition

Definition 1 (SIM-SO-CPA security)

Let PKE be a public key encryption scheme. PKE is SIM-SO-CPA secure if for every PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S} := \mathcal{S}(\mathcal{A})$ such that the distributions induced by

$\mathcal{A}$ run in the *real* game and $\mathcal{S}$ run in the *ideal* game

are computationally indistinguishable.

DHIES

Let g be a generator of a group of size p, $\{0,1\}^\ell$ a message space and $H : \langle g \rangle \mapsto \{0,1\}^\ell$ a hash function.

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$X := g^x$
$pk := (g, p, X, H)$
$sk := x$
Return pk

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$\underline{\text{Enc}_{pk}(m)}$
$r \xleftarrow{\$} \mathbb{Z}_p$
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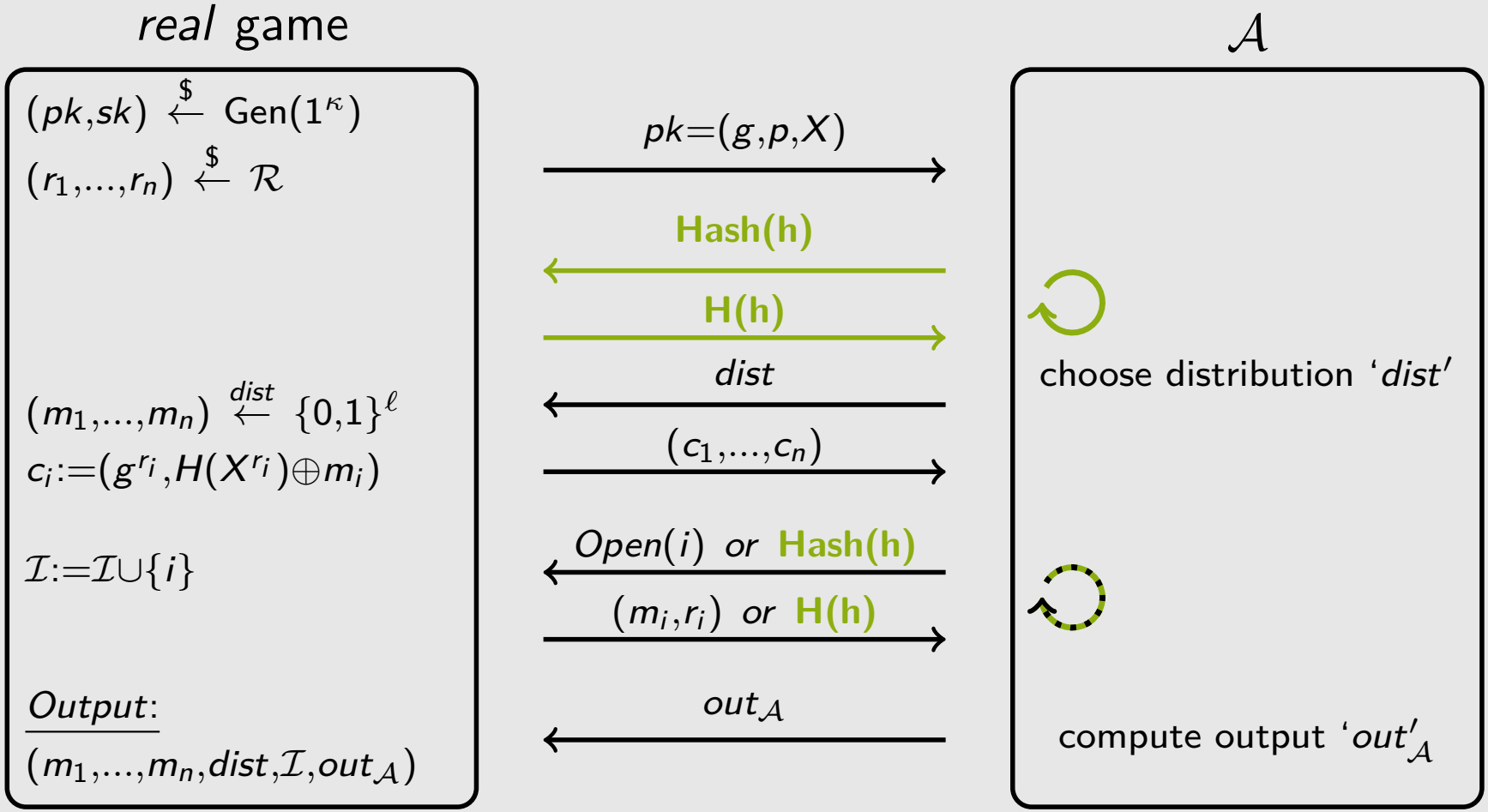
Notice, that we have to provide $\mathcal{A}$ oracle access to H.

SIM-SO-CPA security game for DHIES



Notice, that we sample r_i in advance.

Theorem 2

The DHIES encryption scheme is SIM-SO-CPA secure in the random oracle model, if the CDH assumption holds.

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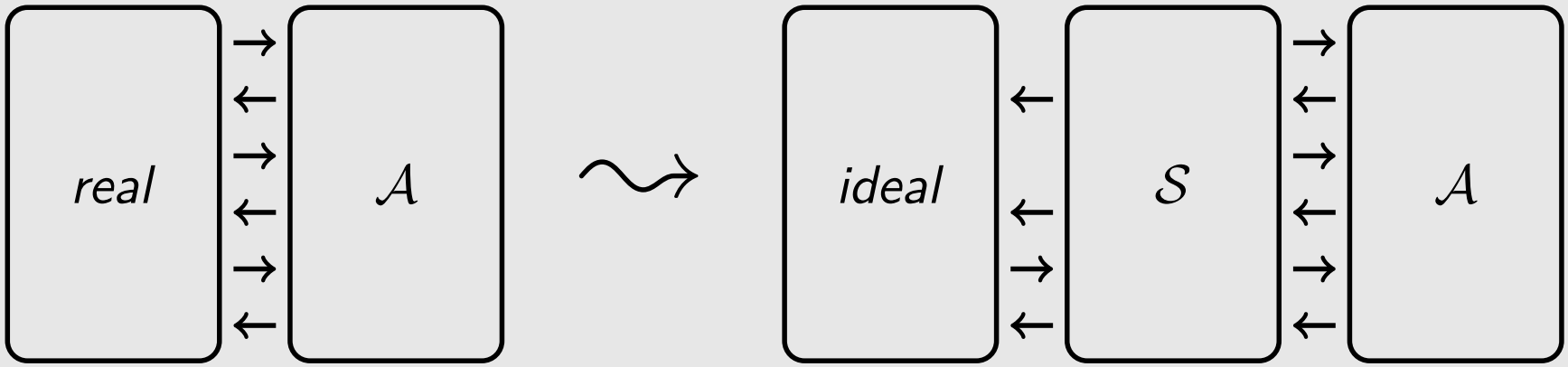

Usual idea: Proceed in a sequence of games until a simulator can take over and run $\mathcal{A}$ on its own.

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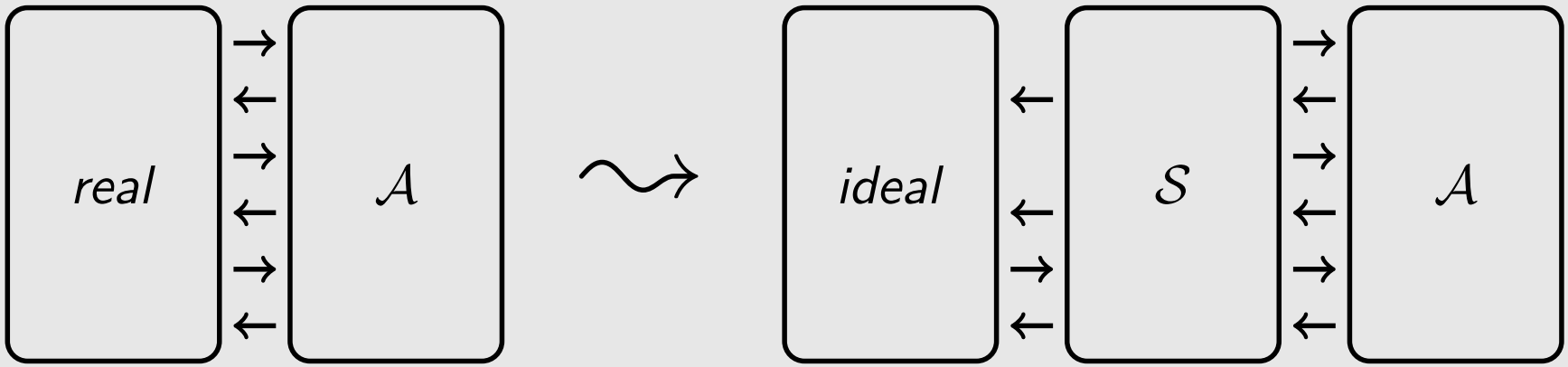
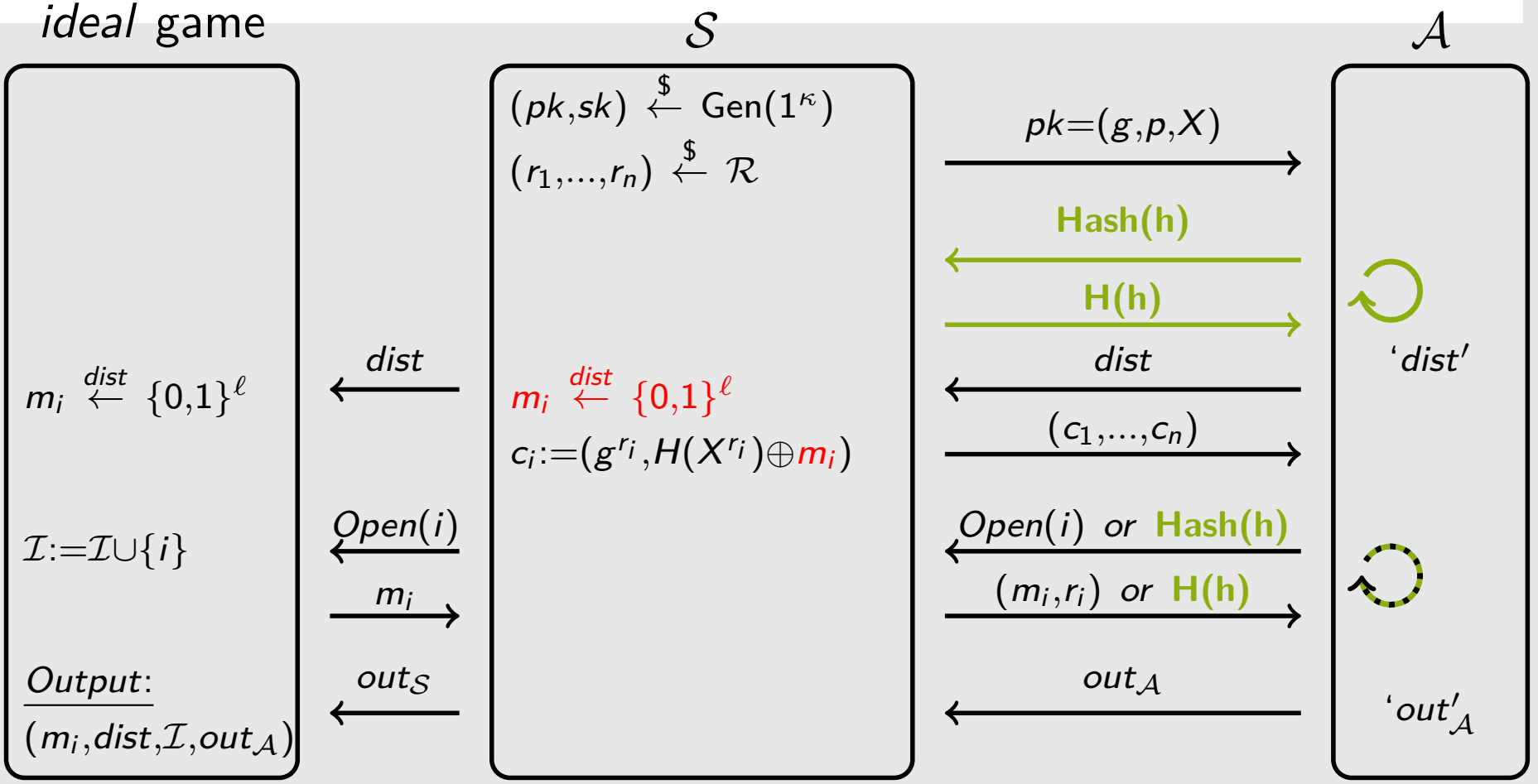


Our approach: We try to construct a simulator right away to see where we run into pitfalls.

Proof.


ideal game
$\mathcal{S}$
$\mathcal{A}$
$(pk,sk) \xleftarrow{\$} \mathrm{Gen}(1^\kappa)$
$(r_1,\ldots,r_n) \xleftarrow{\$} \mathcal{R}$
$pk=(g,p,X)$
Hash(h)
H(h)
'dist'
dist
$m_i \xleftarrow{dist} \{0,1\}^\ell$
$m_i \xleftarrow{dist} \{0,1\}^\ell$
$c_i:=(g^{r_i},H(X^{r_i})\oplus m_i)$
$(c_1,\ldots,c_n)$
$\mathcal{I}:=\mathcal{I}\cup\{i\}$
Open(i)
Open(i) or Hash(h)
m_i
(m_i,r_i) or H(h)
Output:
$(m_i,dist,\mathcal{I},out_\mathcal{A})$
$out_\mathcal{S}$
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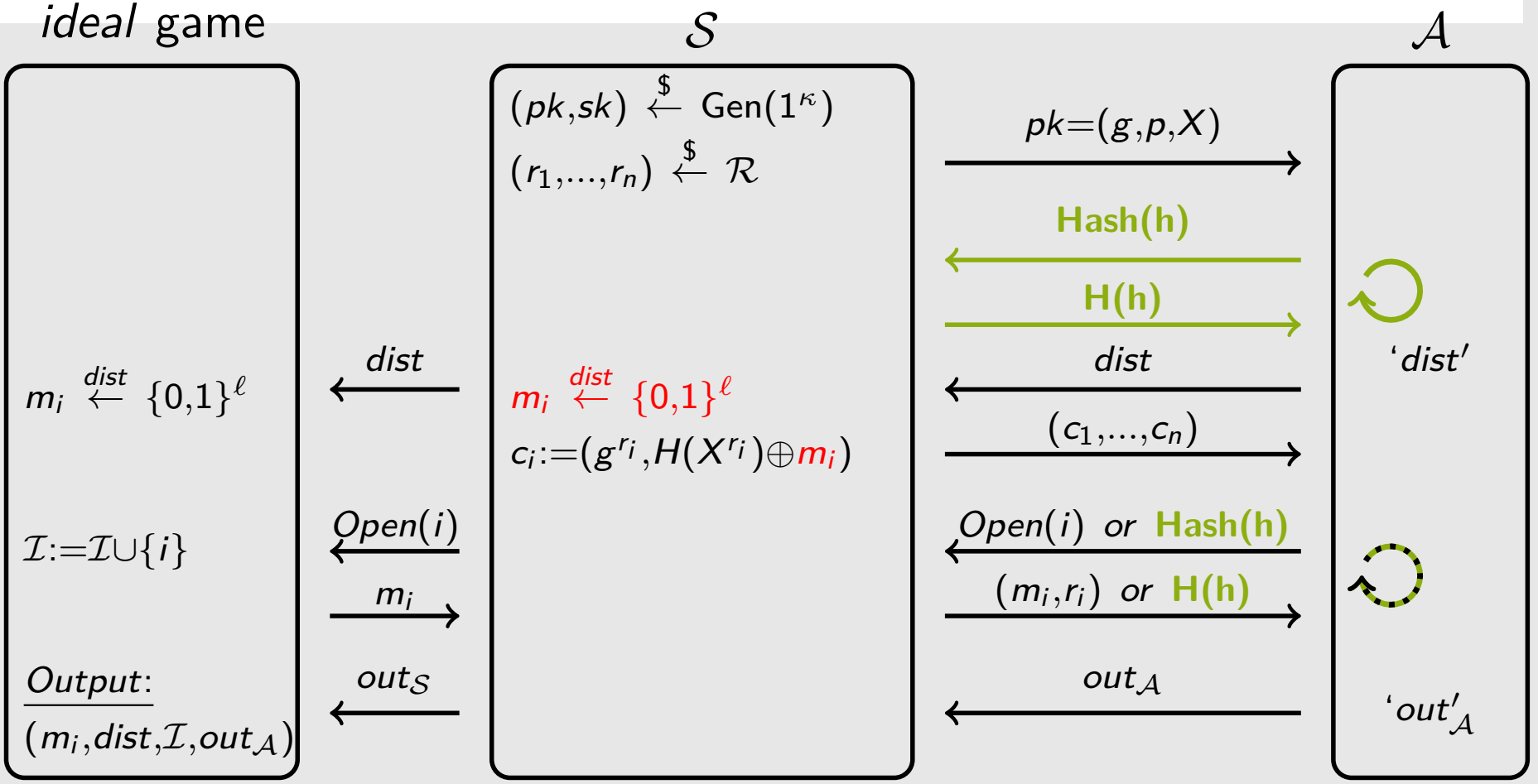
'out'$_\mathcal{A}$

Proof.



Step 1) Make $H(X^{r_i}) \oplus m_i$ uniformly random.

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Abort condition (earlyAbort)

We abort $\mathcal{A}$ if $\mathcal{A}$ should query some $H(X^{r_i})$ before sending dist.

$$\Pr[earlyAbort] \leq negl.$$

Proof.

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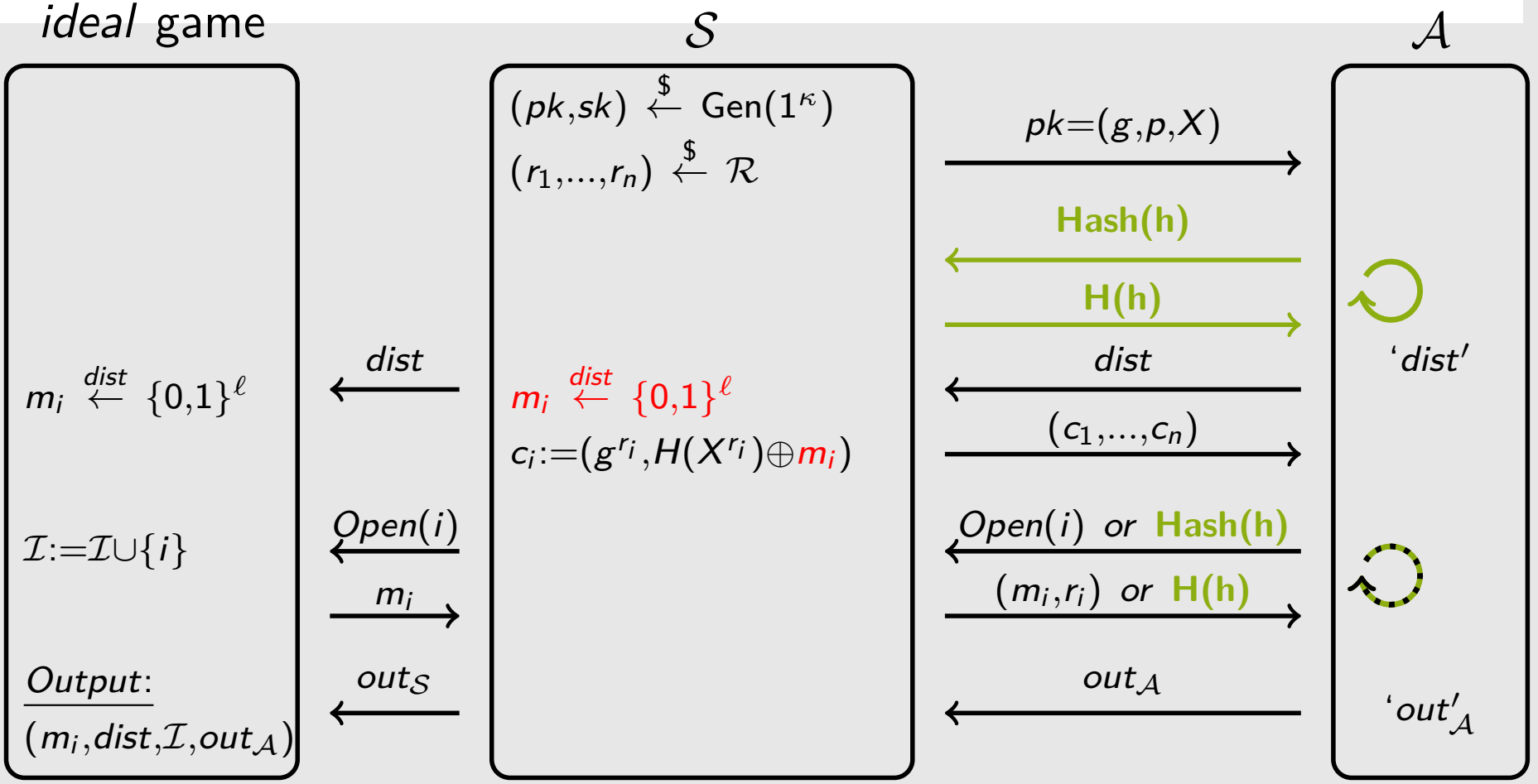
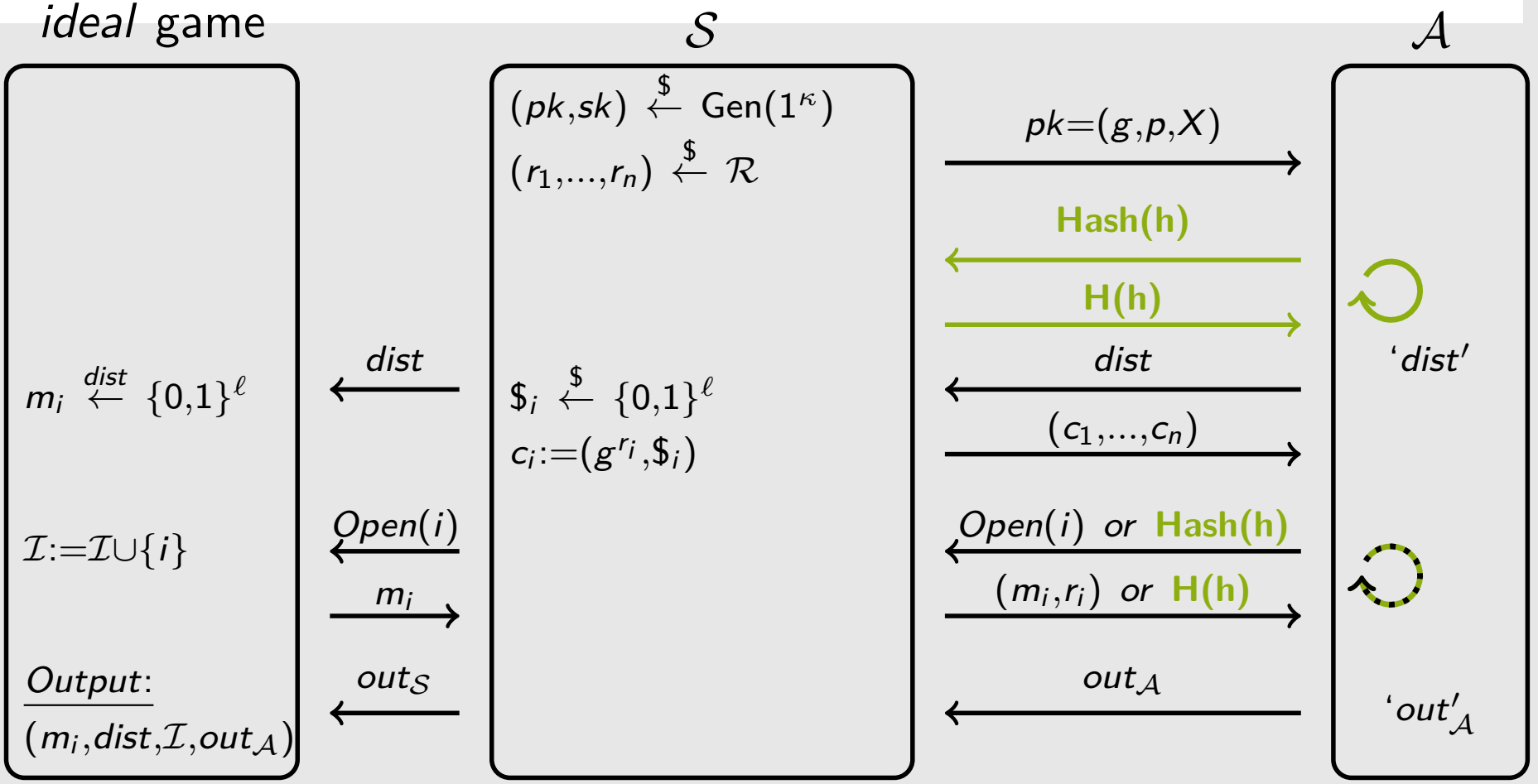
Statistical argument suffices.

Proof.



Step 2) Change encryption

Proof.



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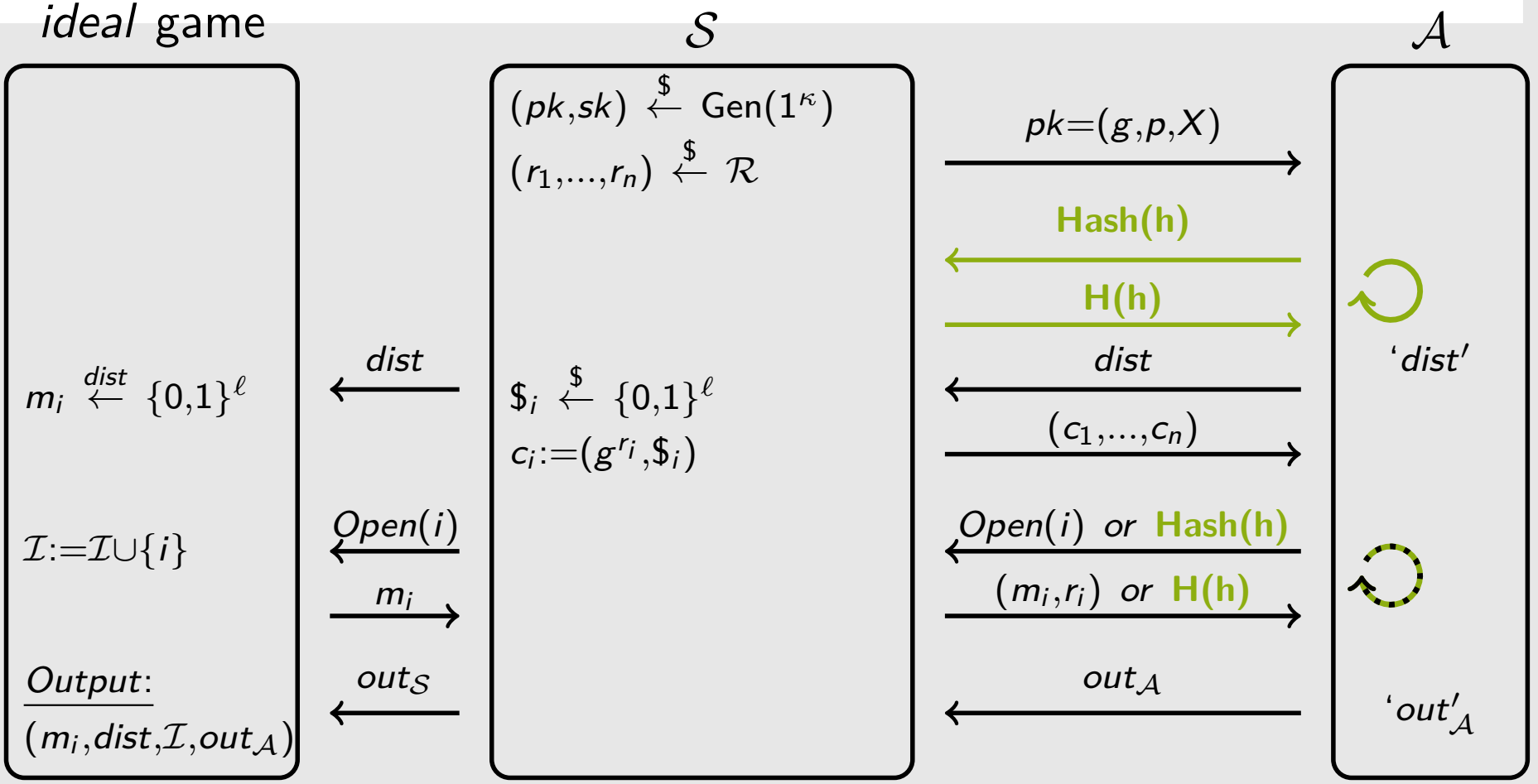
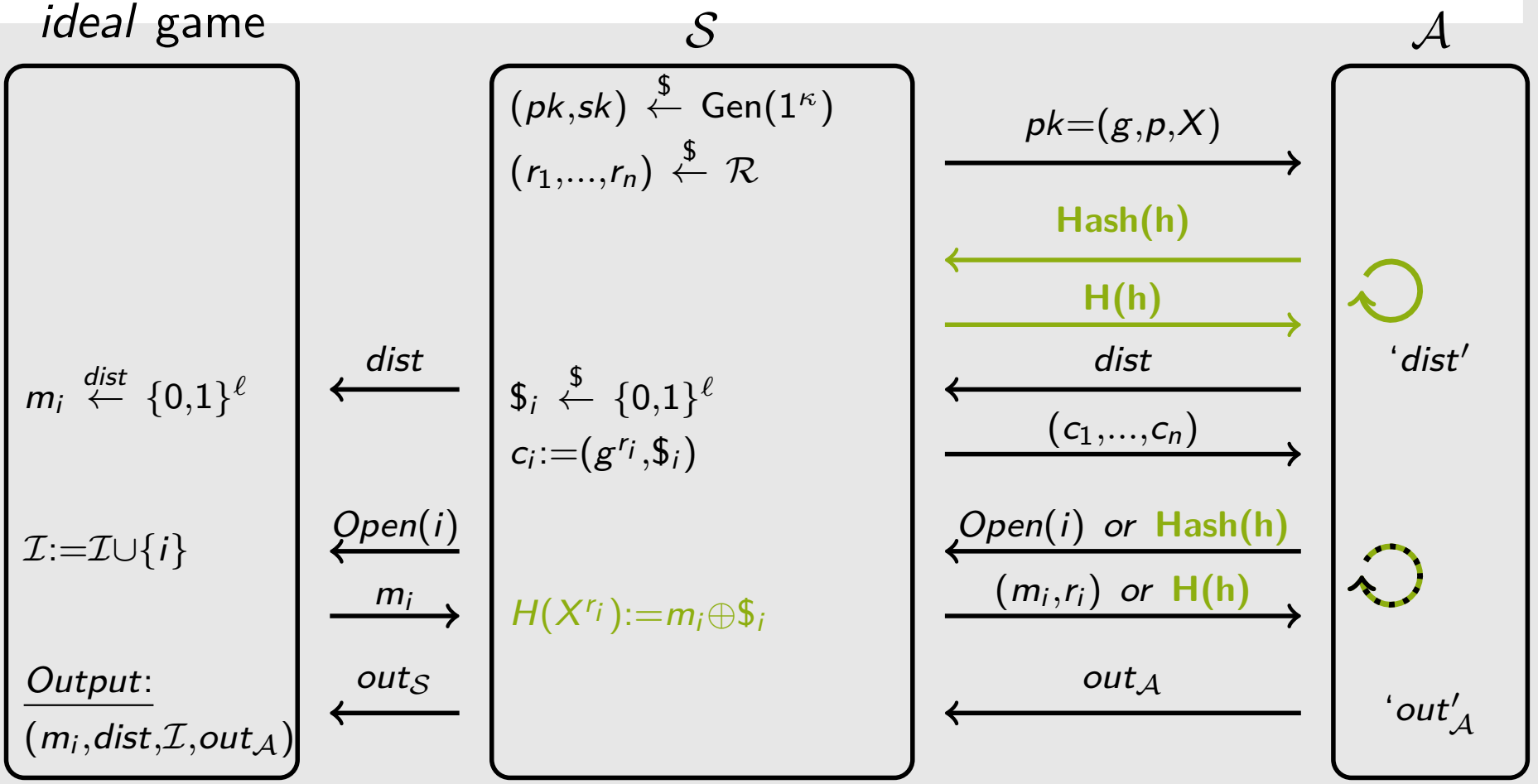


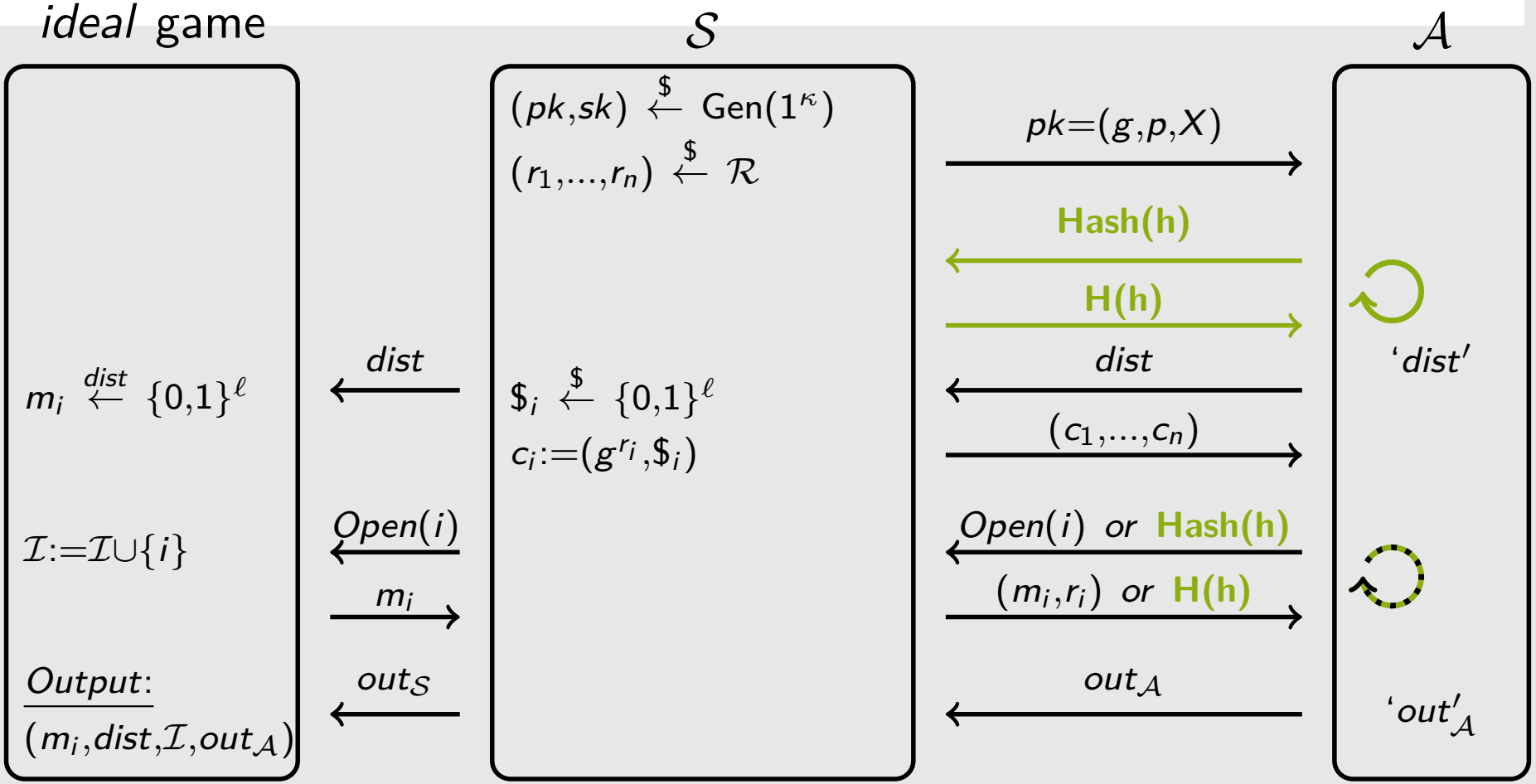
Step 3) How to process **Hash(X^{r_i})** queries?

Proof.



Case 1) $\mathcal{A}$ called $Open(i)$ before querying **Hash(X^{r_i})**.

Proof.



Case 2) $\mathcal{A}$ did not called $Open(i)$ before querying **Hash(X^{r_i})**.

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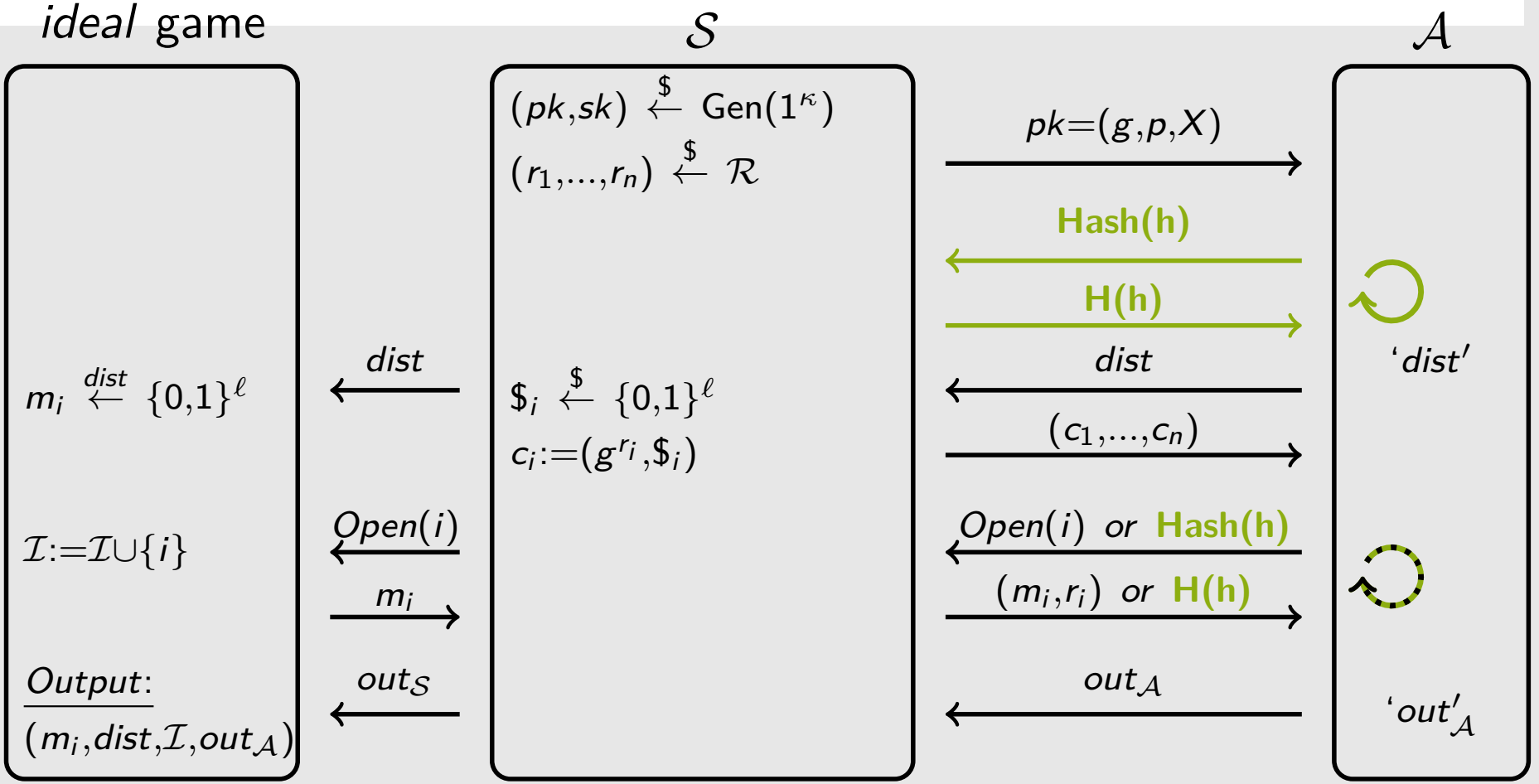

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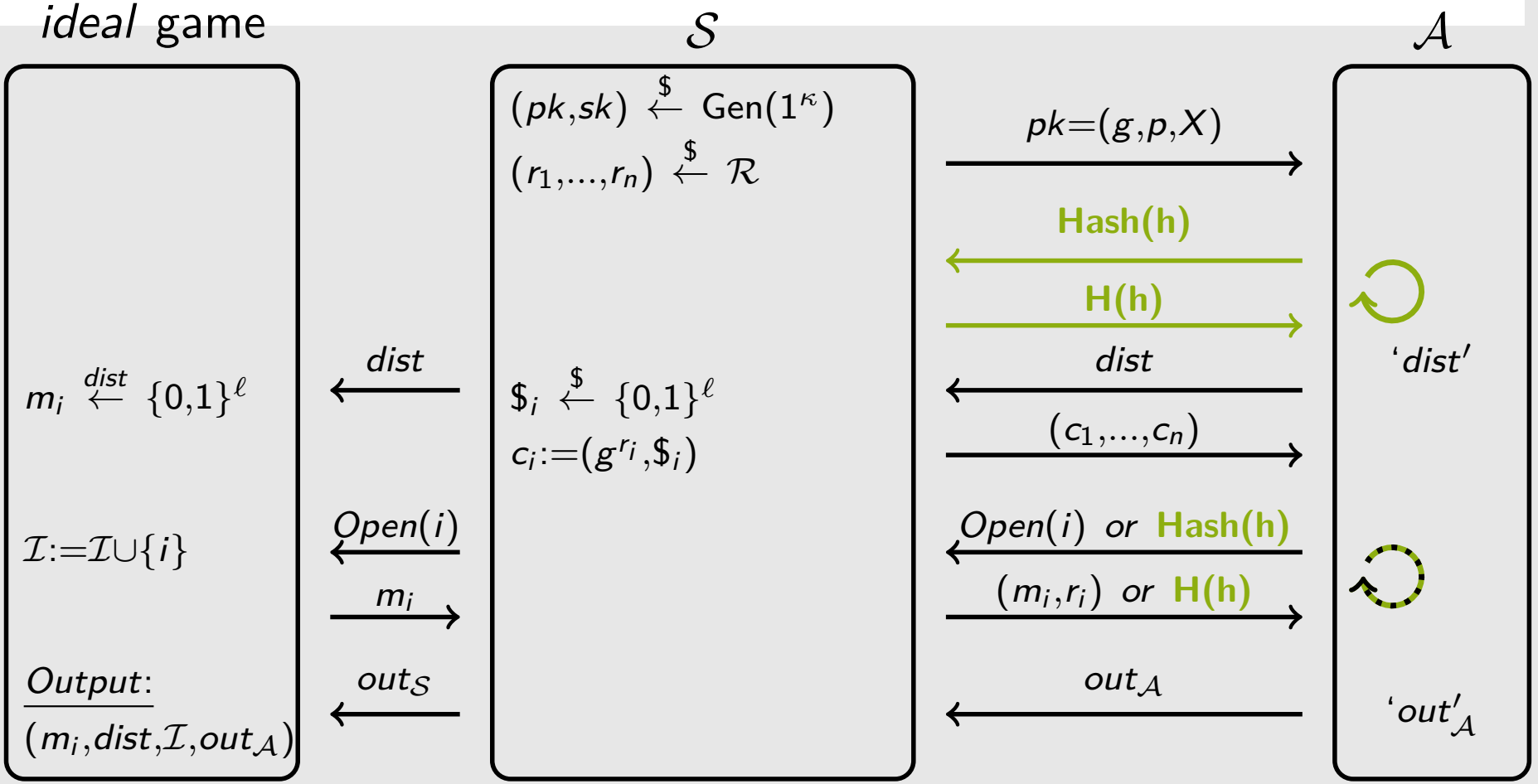


Case 2) $\mathcal{A}$ did not called $Open(i)$ before querying **Hash(X^{r_i})**.
$\mathcal{S}$ can neither call $Open(i)$, nor answer $\mathcal{A}$'s query.

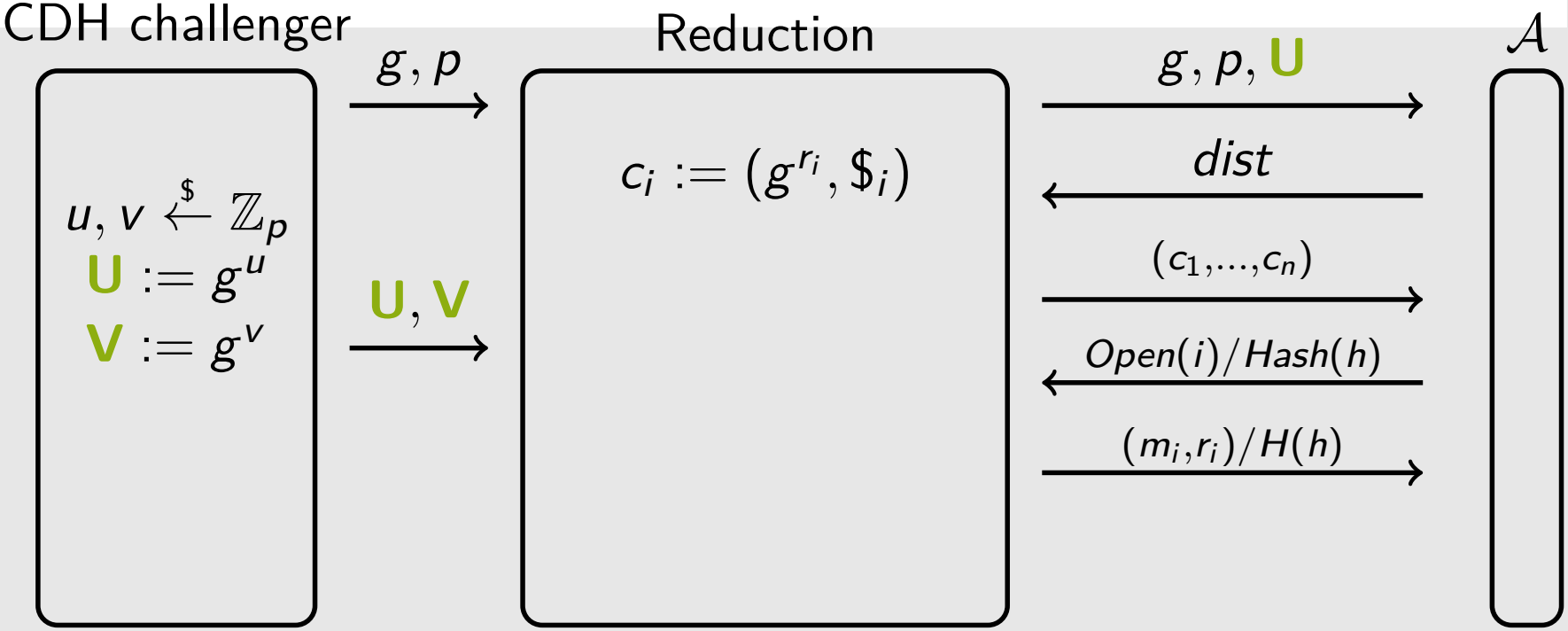
Proof.

Abort condition (AbortH)

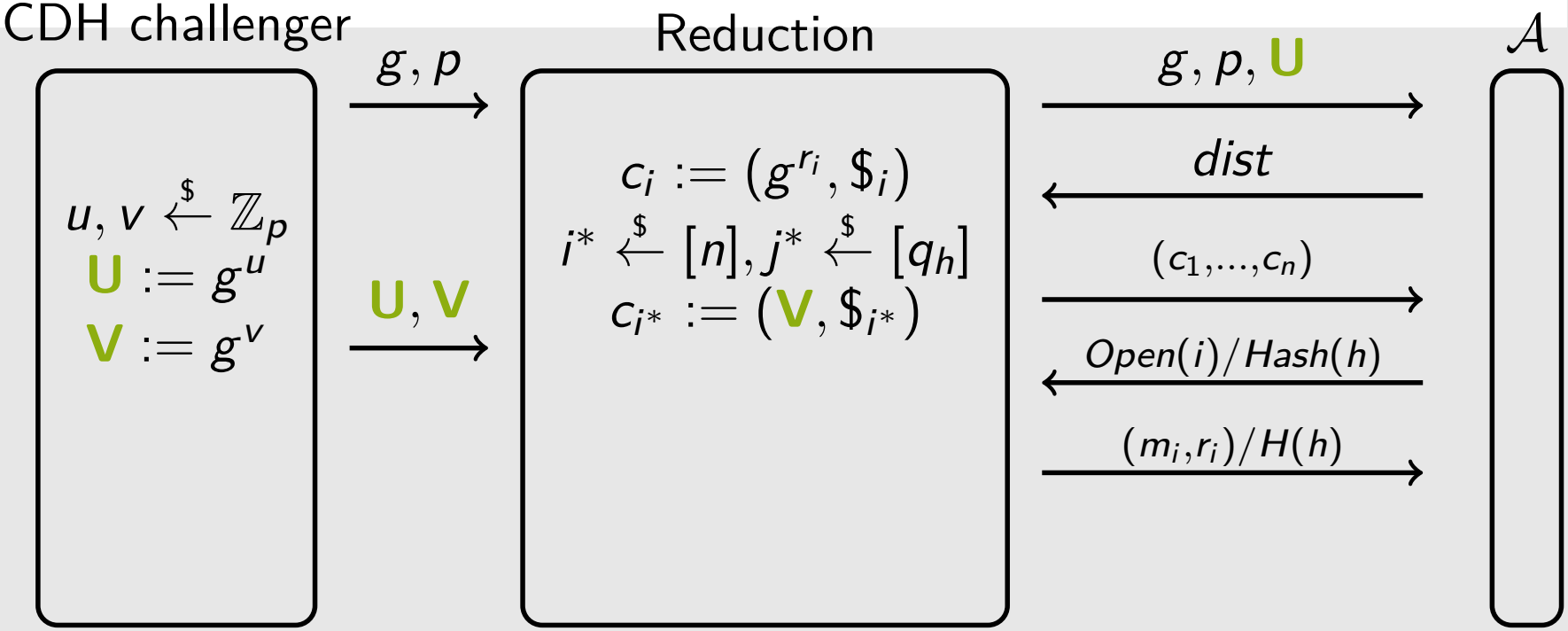
We abort $\mathcal{A}$ if $\mathcal{A}$ calls $H(X^{r_i})$ and did not call $Open(i)$ before.

$$\Pr[AbortH] \leq negl.$$

Proof.


CDH challenger
Reduction
$\mathcal{A}$
g, p
$g, p, \mathbf{U}$
$u, v \xleftarrow{\$} \mathbb{Z}_p$
$\mathbf{U} := g^u$
$\mathbf{V} := g^v$
$\mathbf{U}, \mathbf{V}$
$c_i := (g^{r_i}, \$_i)$
$dist$
$(c_1,\ldots,c_n)$
$Open(i)/Hash(h)$
$(m_i,r_i)/H(h)$

Proof.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$c_i := (g^{r_i}, \$_i)$
$i^* \xleftarrow{\$} [n], j^* \xleftarrow{\$} [q_h]$
$c_{i^*} := (\mathbf{V}, \$_{i^*})$
$g, p, \mathbf{U}$
$dist$
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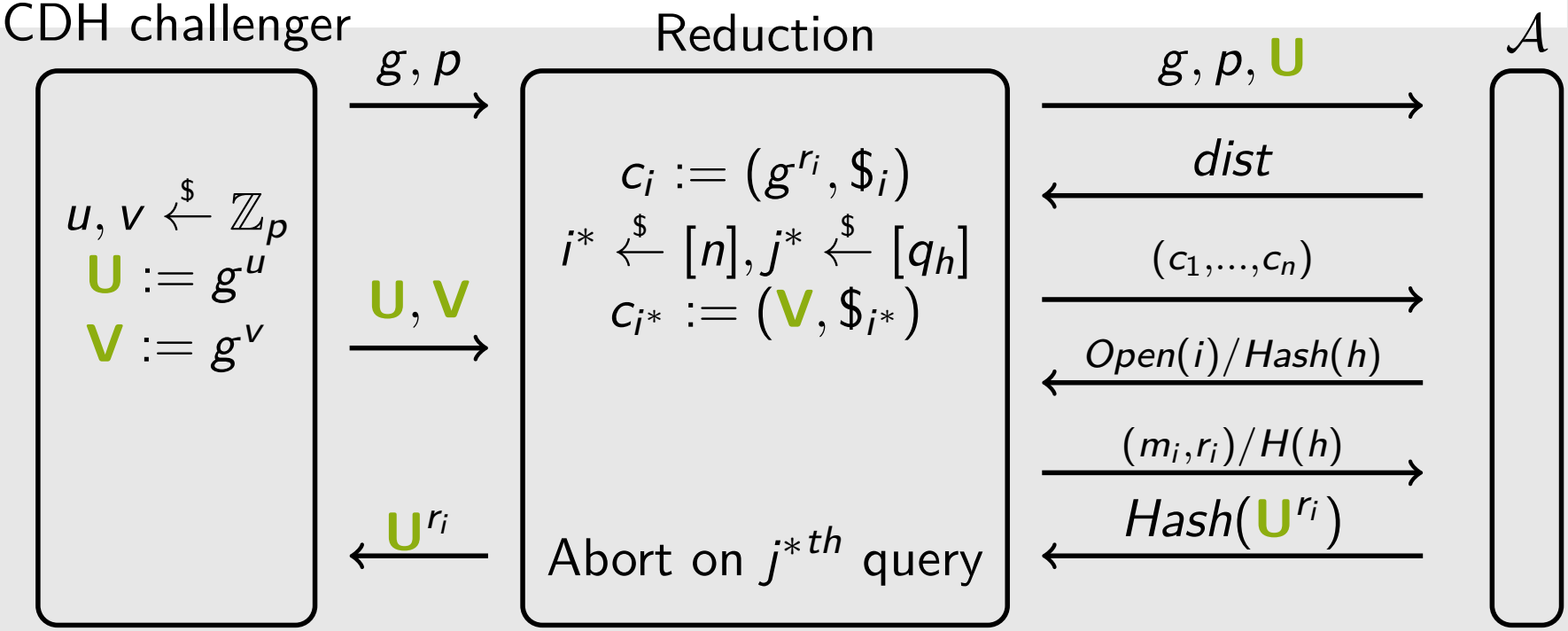

We have to hide our own challenge in the right ciphertext: $1/n$

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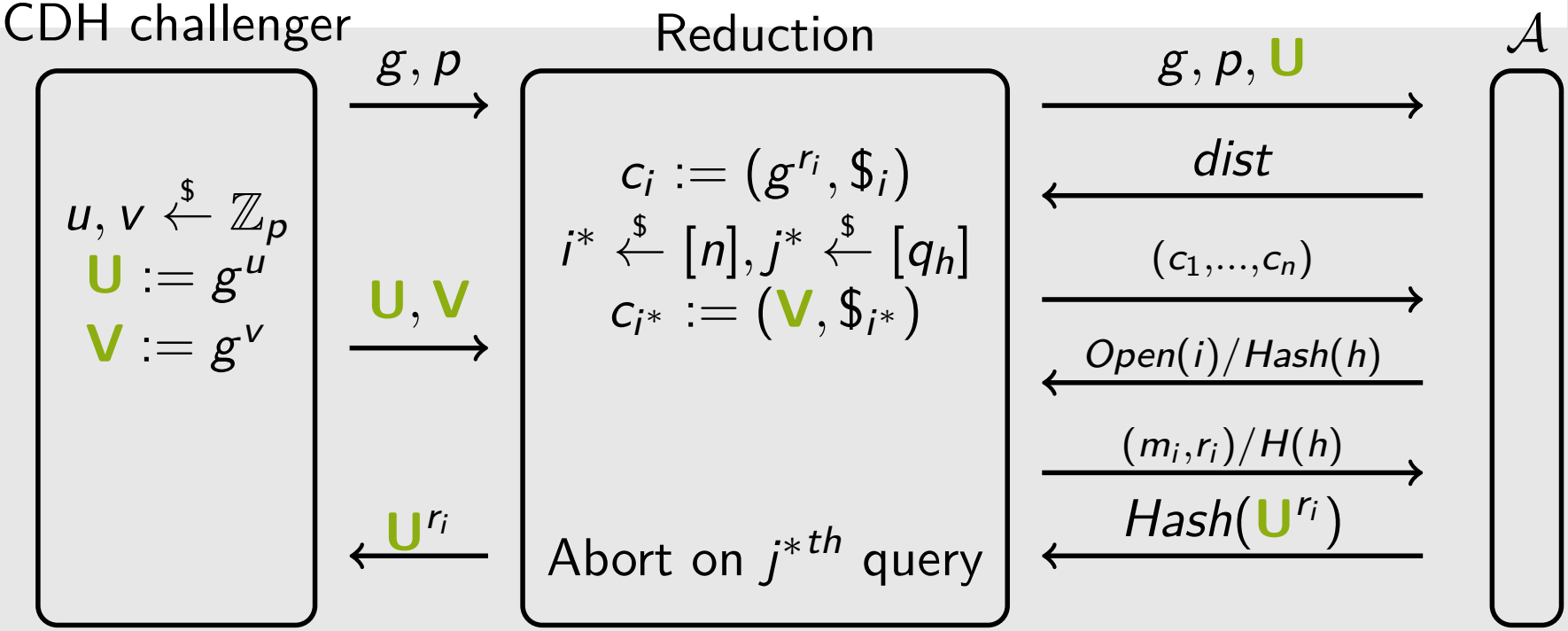

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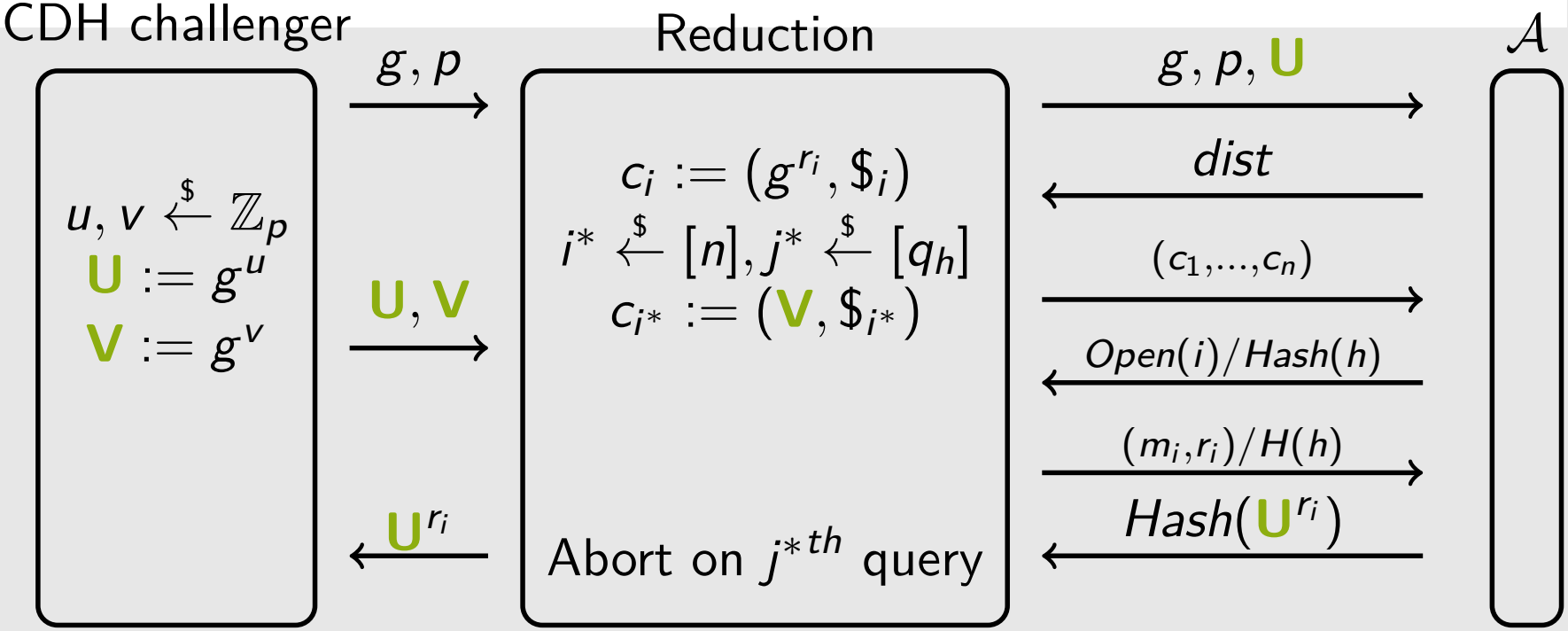

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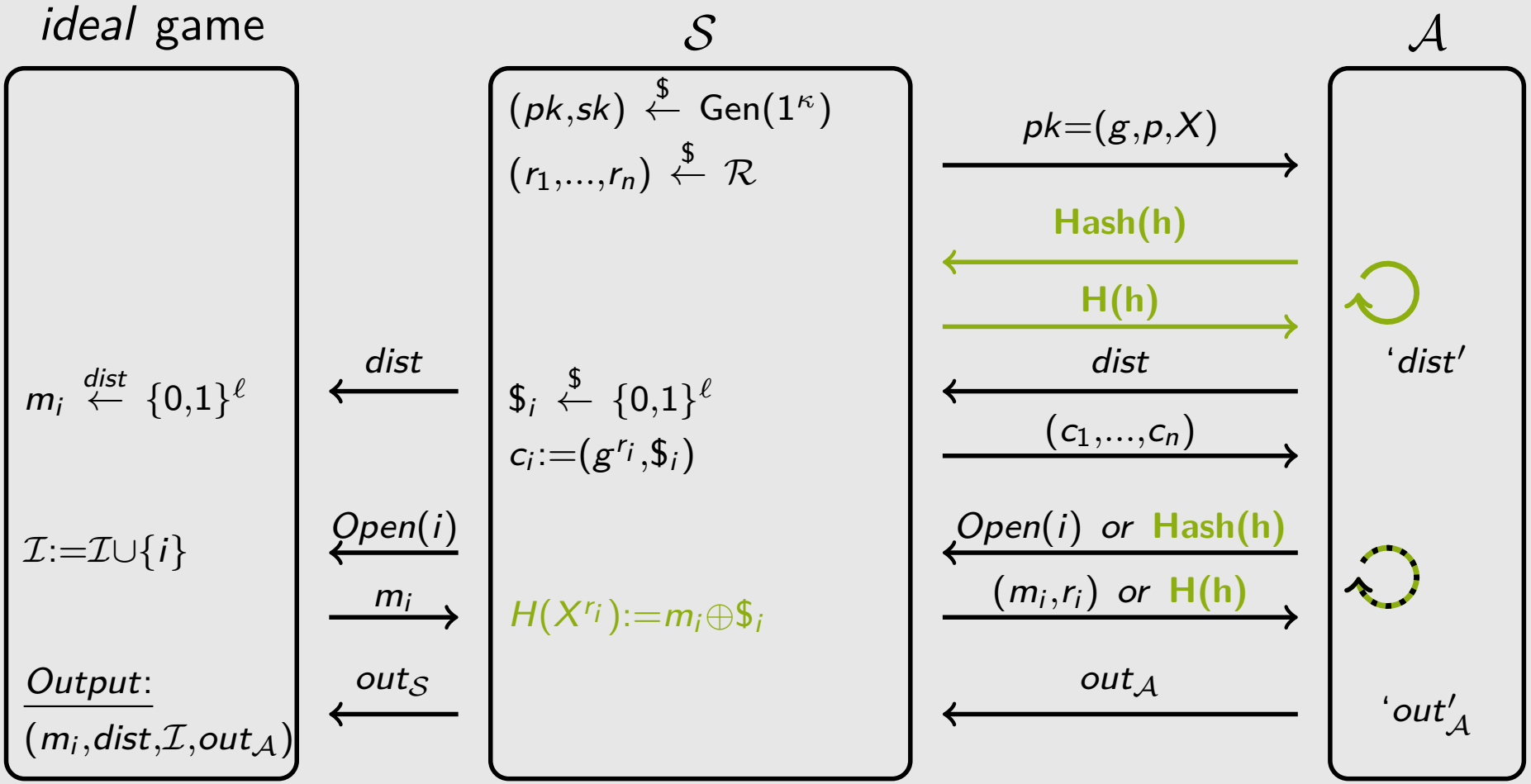
Have to abort on the right query: $1/q_h$

$$\Pr[AbortH] \leq n \cdot q_h \cdot \mathbf{Adv}_g^{\mathrm{CDH}}(\mathcal{B})$$

Proof.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SIM-SO-CCA security for free in the ROM

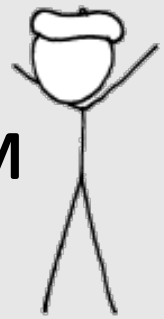

Image source: xkcd.com

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Many thanks for your attention!

QUESTIONS?

Horst Görtz Institut für IT-Sicherheit